# SOLUTIONS TO

Written Reexam at the Department of Economics summer 2020

**Economics of the Environment and Climate Change**

Final Reexam

August 20, 2020

(3-hour open book exam)

Answers only in English.

**This set of solutions to the exam questions consists of 10 pages in total, including this front page.**

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**Written Reexam in the Economics of the Environment and Climate Change, Spring 2020**

**OPTIMAL ENVIRONMENTAL TAX POLICY**

In the following you will be asked to study the optimal environmental tax policy in an economy where the representative consumer consumes a "clean" (non-polluting) good $x_1$ and a "dirty" (polluting) good $x_2$. The consumer also gets utility from a public good *G* provided by the government, whereas the consumer's labour supply *L* and the level of pollution *P* generate disutility. The utility function *U* of the representative consumer takes the following form where $\eta$ and $\varepsilon$ are constant parameters:

$$U = \left(\frac{\eta}{\eta-1}\right) x_1^{\frac{\eta-1}{\eta}} + \left(\frac{\varepsilon}{\varepsilon-1}\right) x_2^{\frac{\varepsilon-1}{\varepsilon}} + g(G) - L - e(P), \tag{1}$$

$$\eta > 1, \quad \varepsilon > 1, \quad g'(G) > 0, \quad g''(G) < 0, \quad e'(P) > 0, \quad e''(P) > 0.$$

The term $-L$ in (1) measures the disutility from work. The function $g(G)$ measures the utility from the public good, and the function $e(P)$ captures the disutility from pollution. The assumptions $g'(G) > 0$ and $g''(G) < 0$ mean that the marginal utility of the public good is positive but decreasing, while the assumptions $e'(P) > 0$ and $e''(P) > 0$ reflect that the marginal disutility from pollution is positive and increasing.

The production and/or consumption of one unit of the dirty good $x_2$ causes emission of one unit of a pollutant. All consumers in the economy are identical, and their total number is *n*, so the total level of pollution is

$$P = nx_2. \tag{2}$$

The number of consumers *n* is very large, so the individual consumer feels unable to influence the total level of pollution *P*. Thus the individual consumer takes *P* as given when making her decisions on consumption and labour supply. All consumers feel the same disutility from pollution, so the pollutant considered is "uniformly mixing".

The production of all the goods $x_1$, $x_2$ and *G* takes place under constant returns to scale with labour as the only input. Without any loss of generality, we can choose our unit for measuring *L* such that the wage rate for one unit of labour is 1. We can also choose the units for measuring the quantities $x_1$, $x_2$ and *G* such that it takes exactly one unit of labour to produce one unit of each good. When the wage rate is 1, the constant marginal cost of producing each good will then also be 1, so under perfect competition the equilibrium prices of each good will likewise be 1. We can therefore write the consumer's budget constraint in the following simple form, where *t* is a pollution tax and *T* is a lump-sum tax levied by the government:

$$x_1 + (1+t)x_2 = L - T. \tag{3}$$

The left-hand side of (3) is total consumer spending on private goods, and the right-hand side is the consumer's disposable income, since *L* is total labour income, given that the wage rate is 1.

The representative consumer wants to maximise her utility (1) subject to the budget constraint (3). The Lagrange function $\ell^c$ corresponding to this problem is

$$\ell^c = \left(\frac{\eta}{\eta-1}\right)x_1^{\frac{\eta-1}{\eta}} + \left(\frac{\varepsilon}{\varepsilon-1}\right)x_2^{\frac{\varepsilon-1}{\varepsilon}} + g(G) - L - e(P) - \lambda\left[x_1 + (1+t)x_2 - L + T\right], \tag{4}$$

where $\lambda$ is the Lagrange multiplier associated with the consumer's budget constraint, and where the consumer takes *P* as well as the government's policy instruments *G*, *t* and *T* as given.

*Question 1.* Show that the first-order conditions for the solution to the consumer's utility maximisation problem combined with the consumer budget constraint imply the following demand functions for the two private goods,

$$x_1 = 1, \qquad x_2 = (1+t)^{-\varepsilon}, \tag{5}$$

and the following labour supply function,

$$L = 1 + (1+t)^{1-\varepsilon} + T. \tag{6}$$

Give a brief economic explanation for the way the tax instruments $t$ and $T$ influence labour supply (Hint: Recall the assumption in (1) that $\varepsilon > 1$). (End of Question 1).

*Answer to Question 1:* Differentiating the Lagrangian (4), we obtain the following first-order conditions:

$$\partial \ell^c / \partial x_1 = 0 \quad \Rightarrow \quad x_1^{-\frac{1}{\eta}} = \lambda, \tag{i}$$

$$\partial \ell^c / \partial x_2 = 0 \quad \Rightarrow \quad x_2^{-\frac{1}{\varepsilon}} = \lambda(1+t), \tag{ii}$$

$$\partial \ell^c / \partial L = 0 \quad \Rightarrow \quad \lambda = 1. \tag{iii}$$

Inserting (iii) in (i) we immediately get the result $x_1 = 1$, and inserting (iii) in (ii) and raising both sides of the resulting equation to the power $-\varepsilon$, we obtain the result $x_2 = (1+t)^{-\varepsilon}$. Finally, we substitute these two results in the consumer budget constraint (3) and isolate $L$ on the left-hand side of the resulting equation to find the result (6). The labour supply function (6) shows that the pollution tax reduces labour supply, given the assumption $\varepsilon > 1$. The reason is that the pollution tax reduces the consumer's real wage by raising the consumer price of $x_2$, and when $\varepsilon > 1$ the resulting negative substitution effect on labour supply dominates the positive income effect. Equation (6) also implies that a higher lump-sum tax stimulates labour supply because it has no substitution effect but only an income effect. This income effect means that the consumer cannot afford to consume as much leisure as before, leading to an increase in labour supply (less leisure). *(End of answer to Question 1).*

When the consumer's optimum conditions (5) and (6) are plugged into the direct utility function (1) along with (2), we obtain the *indirect* utility function which expresses the consumer's maximum attainable level of utility, given the government's choice of the policy instruments $t$, $T$ and $G$.

*Question 2*. Show by using (1), (2), (5) and (6) that the consumer's indirect utility function *V* can be written as

$$V = \frac{1}{\eta - 1} + \frac{(1+t)^{1-\varepsilon}}{\varepsilon - 1} - T + g(G) - e\left(n(1+t)^{-\varepsilon}\right). \tag{7}$$

Explain briefly the ways in which the pollution tax *t* affects the consumer's maximum attainable utility level. (End of Question 2).

*Answer to Question 2:* Inserting the result $x_2 = (1+t)^{-\varepsilon}$ in (2), we find $P = e\left(n(1+t)^{-\varepsilon}\right)$. Substituting this plus (5) and (6) into (1), we get

$$V = \frac{\eta}{\eta - 1} + \left(\frac{\varepsilon}{\varepsilon - 1}\right)(1+t)^{1-\varepsilon} + g(G) - \left[1 + \overbrace{(1+t)^{1-\varepsilon} + T}^{L}\right] - e\left(n(1+t)^{-\varepsilon}\right). \tag{iv}$$

Since

$$\frac{\eta}{\eta - 1} - 1 = \frac{1}{\eta - 1} \quad \text{and} \quad \left(\frac{\varepsilon}{\varepsilon - 1}\right)(1+t)^{1-\varepsilon} - (1+t)^{1-\varepsilon} = \frac{(1+t)^{1-\varepsilon}}{\varepsilon - 1},$$

we can simplify the expression in (iv) to the expression in (7). Equation (7) shows that the pollution tax has two offsetting effects on consumer utility. The term $(1+t)^{1-\varepsilon}/(\varepsilon - 1)$ is the net effect on utility of the higher consumer price of the dirty good which reduces the consumption of that good while increasing the consumption of leisure (reducing labour supply). This net effect on utility is negative, given the assumption that $\varepsilon > 1$. On the other hand, the term $-e\left(n(1+t)^{-\varepsilon}\right)$ reflects that the pollution tax reduces pollution which increases utility. The optimal pollution tax must balance these offsetting effects against each other. *(End of answer to Question 2).*

The government finances its provision of the public good *G* by the revenues from the pollution tax and from the lump sum tax *T* levied on all consumers. Since all consumers are identical and the price (unit cost) of the public good is 1, the government's budget constraint is

$$n(tx_2 + T) = G \quad \Leftrightarrow \quad T = \frac{G}{n} - tx_2. \tag{8}$$

*Question 3*. Insert the consumer's optimal demand for $x_2$ in (8) and use the resulting expression to eliminate *T* from (7) so that the consumer's indirect utility is expressed solely as a function of the two policy variables *t* and *G*.

*Answer to Question 3:* According to (5) the consumer's optimal demand for the dirty good is $x_2 = (1+t)^{-\varepsilon}$. Inserting this into the government budget constraint (8), we get

$$T = \frac{G}{n} - t(1+t)^{-\varepsilon}. \tag{v}$$

Using (v) to eliminate *T* from equation (7), we may write the indirect utility function as

$$V = \frac{1}{\eta - 1} + \frac{(1+t)^{1-\varepsilon}}{\varepsilon - 1} + g(G) - \frac{G}{n} + t(1+t)^{-\varepsilon} - e\left(n(1+t)^{-\varepsilon}\right). \tag{vi}$$

*(End of answer to Question 3).*

*Question 4.* The government chooses its policy instruments *G* and *t* with the purpose of maximising the utility of the representative consumer while obeying the government budget constraint. Use the indirect utility function derived in Question 3 to derive the first-order conditions for the government's optimal choice of *G* and *t*. Explain the economic intuition behind the first-order condition for the optimal choice of *G*. Rewrite the first-order condition for the optimal value of *t* to isolate *t* on the left-hand side of the equation so you get a simple expression for the optimal pollution tax. Explain the economic intuition behind this expression. (Hints: Note that the government budget constraint is already embodied in your expression for *V* derived in Question 3, so you can derive the optimal values of *G* and *t* by maximising this expression without having to set up a Lagrangian function. Further, when stating your final expression for the optimal value of *t*, you may use the fact that $e'\left(n(1+t)^{-\varepsilon}\right) = e'(P)$.)

*Answer to Question 4:* The first-order conditions for the maximisation of (vi) with respect to *G* and *t* are

$$\frac{\partial V}{\partial G} = 0 \quad \Rightarrow \quad ng'(G) = 1, \tag{vii}$$

$$\frac{\partial V}{\partial t}=0 \quad \Rightarrow \quad -(1+t)^{-\varepsilon}+(1+t)^{-\varepsilon}-\varepsilon t(1+t)^{-\varepsilon-1}+e'\left(n(1+t)^{-\varepsilon}\right)\varepsilon n(1+t)^{-\varepsilon-1}=0. \qquad \text{(viii)}$$

The optimum condition (vii) is a version of the Samuelson condition for the optimal supply of a public good. The left-hand side of (vii) measures the total marginal benefits from the public good, summed over all consumers in the economy since a public good is non-rival in consumption, meaning that one person's consumption of the good does not prevent any other consumer from consuming a similar amount of the good. The right-hand side of (vii) is the marginal cost of providing the public good which is 1 in our simple model where we have assumed that the marginal cost of production for each good is constant and equal to one (by our choice of units).

In condition (viii) for the optimal choice of *t* the first two terms on the left-hand side obviously cancel each other. Using the fact that $e'\left(n(1+t)^{-\varepsilon}\right)=e'(P)$, we can therefore rewrite (viii) as

$$-\varepsilon t(1+t)^{-\varepsilon-1}+e'(P)\varepsilon n(1+t)^{-\varepsilon-1}=0 \quad \Leftrightarrow$$

$$t=ne'(P). \qquad \text{(ix)}$$

The right-hand side of (ix) is the marginal external cost that the individual consumer imposes on the rest of society when she increases her consumption of the dirty good by one unit. Since a one unit increase in the consumption of $x_2$ increases emissions by one unit, the external cost takes the form of an increase $e'(P)$ in the disutility of pollution which will be felt by all the *n* consumers in the economy, so $ne'(P)$ is the total marginal external cost of pollution. According to (ix) the optimal pollution tax fully internalizes this externality by imposing a unit tax on the dirty good which exactly equals the marginal environmental cost of consuming the good. This is the environmental tax rule prescribed by Pigou.

(*Note: The following alternative interpretation of the pollution tax rule* (ix) *is quite advanced and is not required for a satisfactory answer to Question 4*). The cleaner environment secured by the pollution tax is a particular form of a public good since the improvement of environmental quality benefits all consumers. Eq. (ix) may therefore be seen as a Samuelson condition for the optimal provision of a public good called “a cleaner environment”. The right-hand side of (ix) measures the total marginal benefits from this good, summed over all consumers, and the left-hand side ($t$) measures the marginal social cost of providing a cleaner environment. To see this, note from (ii) and (iii) that the consumer price $1+t$ of the dirty good measures the monetary value of the marginal utility $x_2^{\frac{-1}{\varepsilon}}$ from consuming this good (denoted by *MU*), while the producer price of the good (i.e., the social cost of producing it, denoted by *PP*) is just 1. The *non-environmental* marginal social cost of giving up one unit of consumption of the dirty good is the difference between the non-environmental marginal benefit from consumption of the dirty good (*MU*) and its marginal cost of production (*PP*). This difference is $MU - PP = 1+t-1 = t$. Since reducing the consumption of the dirty good by one unit also reduces emissions by one unit, the pollution tax rate $t$ is thus the marginal non-environmental cost of improving the environment. When this marginal cost (the left-hand side of (ix)) equals the marginal benefit from a cleaner environment (the right-hand side of (ix)), the government has optimized environmental policy. *(End of answer to Question 4).*

*Question 5.* Now suppose that the economy actually consists of many heterogeneous consumers with different earnings capabilities resulting in an unequal distribution of income. Suppose further that the government cannot impose individualized non-distortionary lump sum taxes to correct for undesirable inequalities in income distribution but that it can impose a progressive income tax. Discuss whether a government in such a society would want to stick to the rule for the optimal pollution tax that you derived in Question 4. (Hint: You are not asked to do any formal mathematical analysis here; a verbal discussion suffices).

*Answer to Question 5:* There is no single “objectively correct” answer to this question, so an answer that reflects coherent economic reasoning and makes intuitive sense is satisfactory.

A natural starting point for the discussion is that the pollution tax rule (ix) in Question (4) is a pure efficiency rule that does not incorporate any concern about income distribution (which is natural since the underlying model assumes that all consumers are identical). When consumers differ in their earnings capacity so that the income distribution is unequal, a utilitarian government will be concerned about the resulting unequal distribution of welfare. However, one could argue that the government should use the progressive income tax to achieve the desired distribution of income and

raise the necessary revenue and then stick to the efficiency rule (ix) to secure an efficient reduction of pollution. This is a sensible answer to the question.
An advanced answer may point out that the progressive income tax will distort labour supply and that the pollution tax will exacerbate this distortion by further eroding the disposable real wage. Hence it might be that the government would want to keep the pollution tax rate somewhat below the Pigouvian level prescribed by (ix) in order not to exacerbate the pre-existing income tax distortion to labour supply too much even if this implies a higher level of pollution than the first-best Pigouvian level. This would also be a thoughtful satisfactory answer to the question.

Finally, if for some reason the government cannot freely design the income tax schedule so as to attain all the income redistribution it would prefer, the government may want to consider the distributional effects as well as the environmental effect when choosing the level of the pollution tax rate. Specifically, if the pollution tax is estimated to be regressive, say, because expenditures on the dirty good (which could be fossil fuel) make up a relatively larger share of the budgets of poorer families, the government might want to keep the pollution tax rate below the Pigouvian level for distributional reasons (and vice versa if the pollution tax is deemed to be progressive). This would also be an acceptable answer to the question.
*(End of answer to Question 5).*

*Question 6*. Environmental economists usually assume that the objective of pollution tax policy is to achieve economic efficiency in resource allocation. Discuss briefly some other objectives that might also be legitimate and relevant targets for policies against pollution.

*Answer to Question 6:* Again, there is no single “objectively correct” answer to this question, but the textbook by Perman et al. (2011) mentions the following alternative targets for pollution policy:

a) *Sustainability and ecological goals*. For ethical reasons, voters and policy makers may want to set targets for pollution that would avoid a decline in biodiversity and/or preserve certain unique ecosystems. This could be seen as especially important if environmental damage to the ecosystems is deemed to be irreversible beyond certain “tipping points”. If there is uncertainty about these tipping points, the Precautionary Principle of environmental policy may call for a tighter pollution target than the one suggested by a standard cost-benefit analysis like the one underlying the pollution tax rule (ix). However, even if the pollution target is set on the basis of sustainability and ecological goals, the environmental economist could still provide useful advice on cost-effective ways to meet the target, say, via environmental taxes or tradeable pollution permits.

b) *Human health protection*. In the areas of air pollution and water pollution, the pollution targets are often set with the purpose of protecting public health by keeping the concentration of pollutants at levels that do not imply “unacceptable” risks to public health. These targets are rarely identified through systematic cost-benefit analysis, although they may to some extent be influenced by the costs of reducing pollution. This approach to the setting of pollution standards reflects the notion that one particular cost of pollution – i.e., its damaging impact on human health – is particularly important and should therefore be decisive for standard setting.

c) *Public preferences*. Points a) and c) are examples of how particular public preferences (possibly expressed through voting processes) may legitimately affect pollution policy. Another such example could be the public’s notion of distributional fairness which could lead to pollution targets that reflect a concern for the distributional impact of environmental policy. For example, some NGOs and policy makers might favour environmental policies that tend to redistribute income from rich to poor and might be sceptical of the use of environmental taxes or other environmental policy instruments if these are deemed to be regressive. However, as discussed in the answer to Question 5, environmental policy may not be an efficient way of achieving income redistribution when the government has access to other policy instruments such as a progressive income and/or targeted public transfers to disadvantaged groups in society.

*(End of answer to Question 6).*

*Note: Question 4 is the most demanding question in this exam. The quality of the answer to Question 4 should therefore carry more weight in the grading than the quality of the answers to the other questions.*